DUALITIES IN TRIANGULATED CATEGORIES
ASSIGNMENT 1. DUE: 27TH MARCH 2024

(1) Let T be a pretriangulated category. Suppose that $T = (X \xrightarrow{f} Y \xrightarrow{g} Z \xrightarrow{h} \Sigma X)$ and $T' = (X' \xrightarrow{f'} Y' \xrightarrow{g'} Z' \xrightarrow{h'} \Sigma X')$ are candidate triangles. Show that if their sum

$$X \oplus X' \xrightarrow{f \oplus f'} Y \oplus Y' \xrightarrow{g \oplus g'} Z \oplus Z' \xrightarrow{h \oplus h'} \Sigma X \oplus \Sigma X'$$

is a triangle, then T and T' are triangles.

(2) (i) Show that the short exact sequence $0 \to \mathbb{Z}/2 \xrightarrow{\cdot 2} \mathbb{Z}/4 \xrightarrow{\text{mod } 2} \mathbb{Z}/2 \to 0$ of $\mathbb{Z}$-modules does not give rise to a triangle in $\mathsf{K}(\mathbb{Z})$.

(ii) Let R be a ring. Prove that if $0 \to L \xrightarrow{f} M \xrightarrow{g} N \to 0$ is a split exact sequence of R-modules, then there is a map $h \colon N \to \Sigma L$ such that $L \xrightarrow{f} M \xrightarrow{g} N \xrightarrow{h} \Sigma L$ is a triangle in $\mathsf{K}(R)$.